Bose Condensate Dynamics

Mean Field and Beyond

Cold Atom Theory

- Mark Fromhold
- Robin Scott
- Bo Xiong
- Tom Judd
- German Sinuco
- Sarah Naylor
- Dmitri Umnov
- Keith Benedict
- Carlos Lobo
- Tao Yang
- Joe Shearring
- Abdul Mohamad
- Samantha Hooley
- Shaun Reynolds

Overview

- Ultra Cold Atoms

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- Truncated Wigner Approximation - Task Farming

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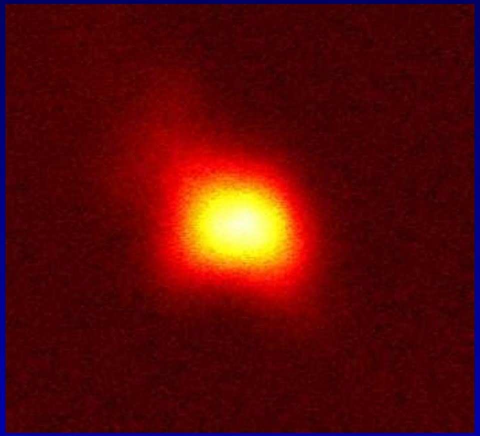
- Confine vapour of alkali metal atoms in magnetic/optical trap



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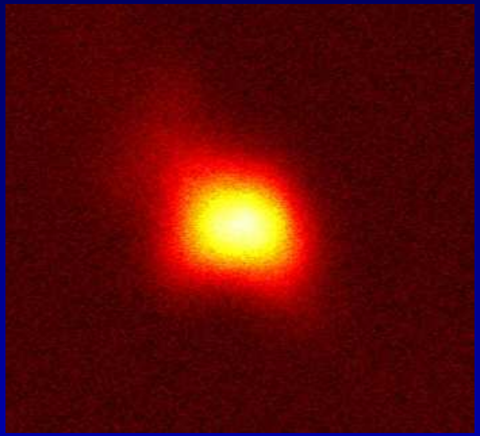

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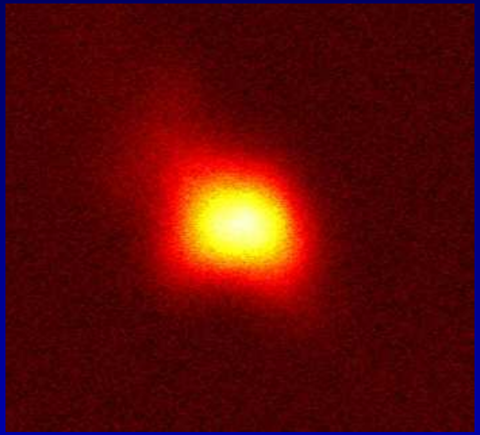

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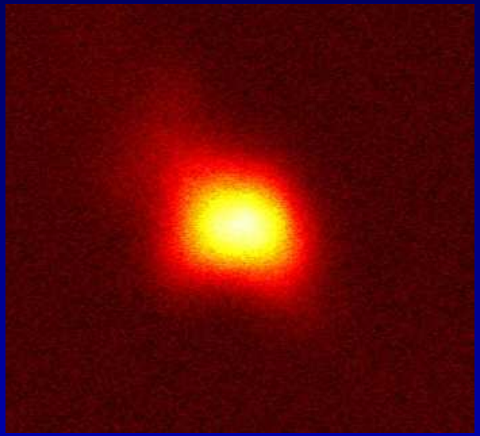
- Lifetime of cold cloud $\sim 1 - 30s$



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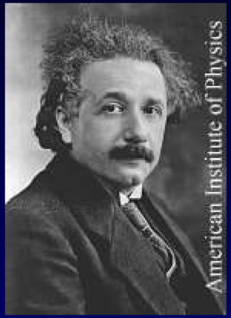

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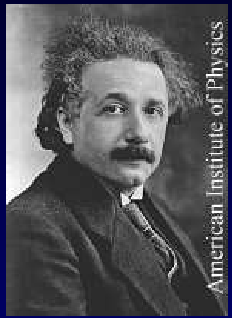

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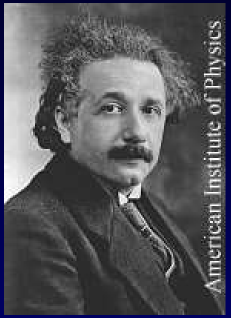

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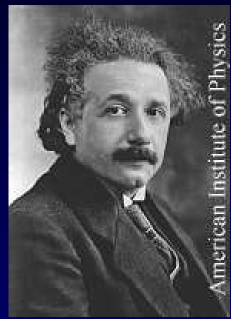

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- Interacting Bose condensed system = superfluid

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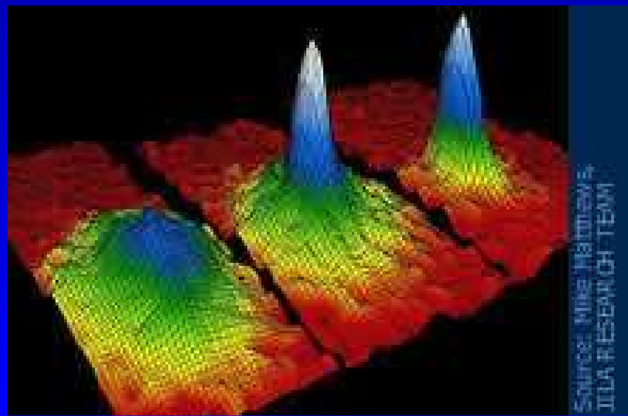

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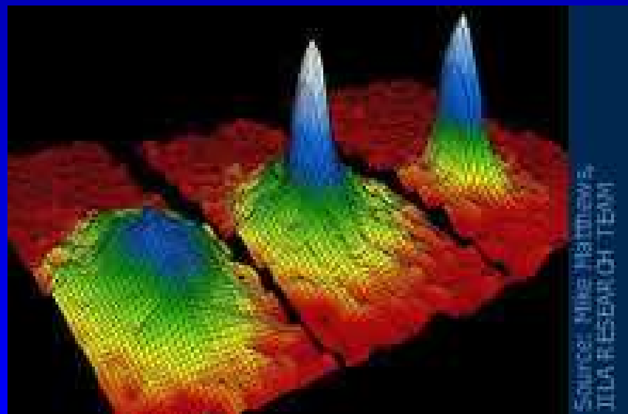

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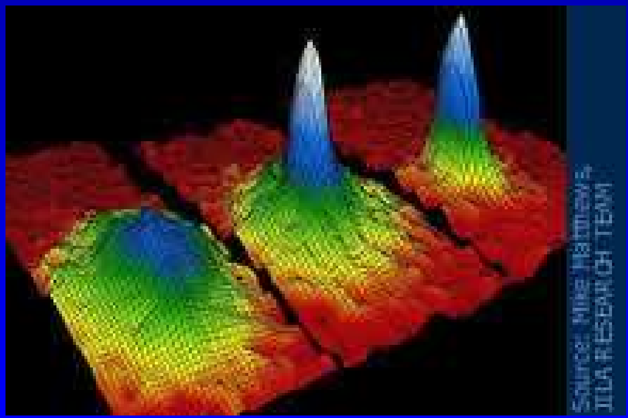

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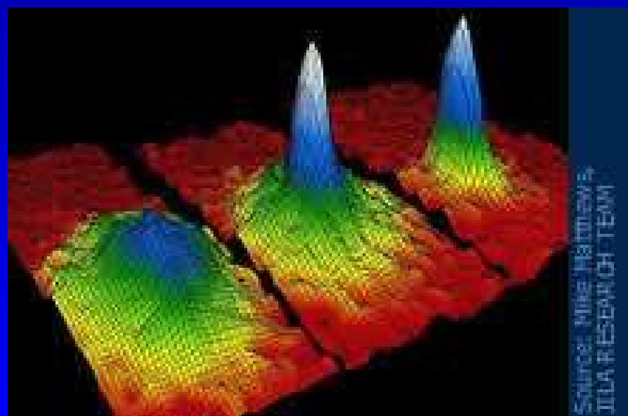

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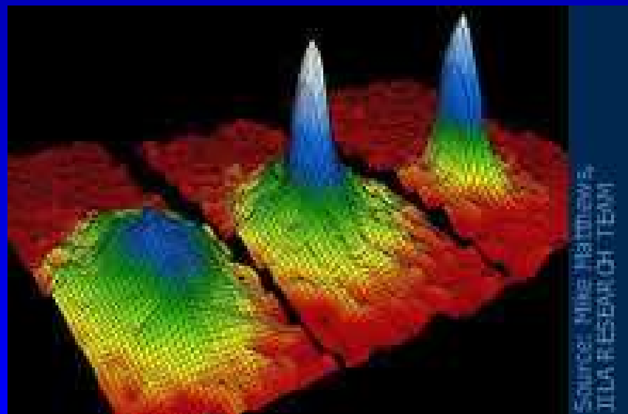
- Time evolution of wavefunction given by time-dependent Gross-Pitaevskii (GP) equation



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- Preserves normalization

Conserved Quantities

- GP Dynamics conserves energy

$$E\left[\phi\left(\mathbf{r}\right)\right] = \int\left\{\frac{\hbar^2}{2m}\left|\nabla\phi\left(\mathbf{r}\right)\right|^2 + V\left(\mathbf{r}\right)\left|\phi\left(\mathbf{r}\right)\right|^2 + \frac{1}{2}\frac{4\pi\hbar^2 a_s}{m}\left|\phi\left(\mathbf{r}\right)\right|^4\right\} d^3\mathbf{r}$$

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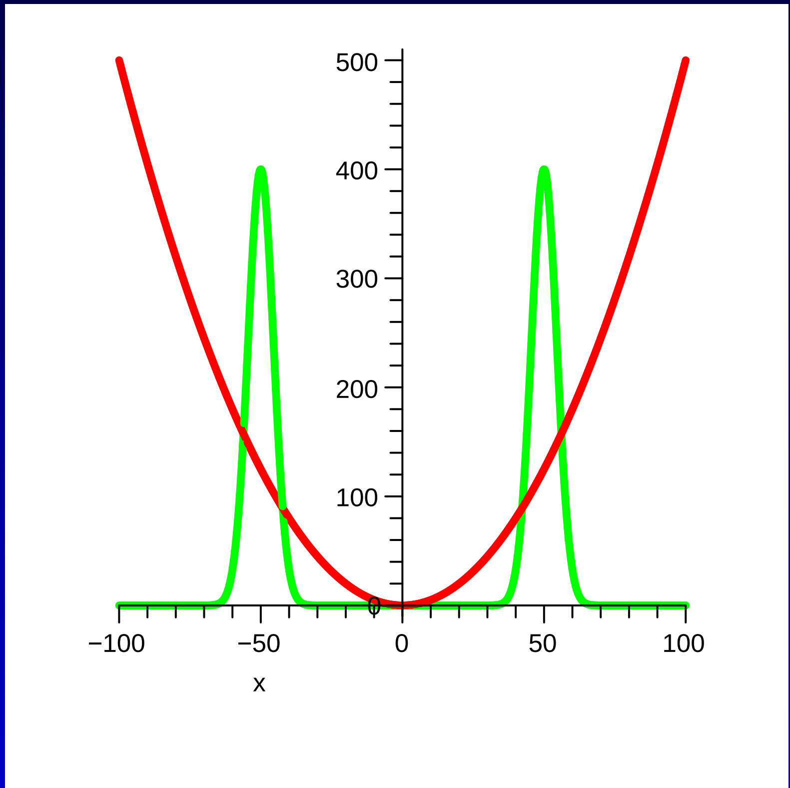
- GP equation then arises from variational principle

$$i\hbar\frac{\partial\phi(\mathbf{r},t)}{\partial t} = \frac{\delta E\left[\phi(\mathbf{r})\right]}{\delta\phi^*(\mathbf{r})}$$

Example - Colliding Clouds

- Consider a condensate inititially split into two coherent components in a parabolic trap



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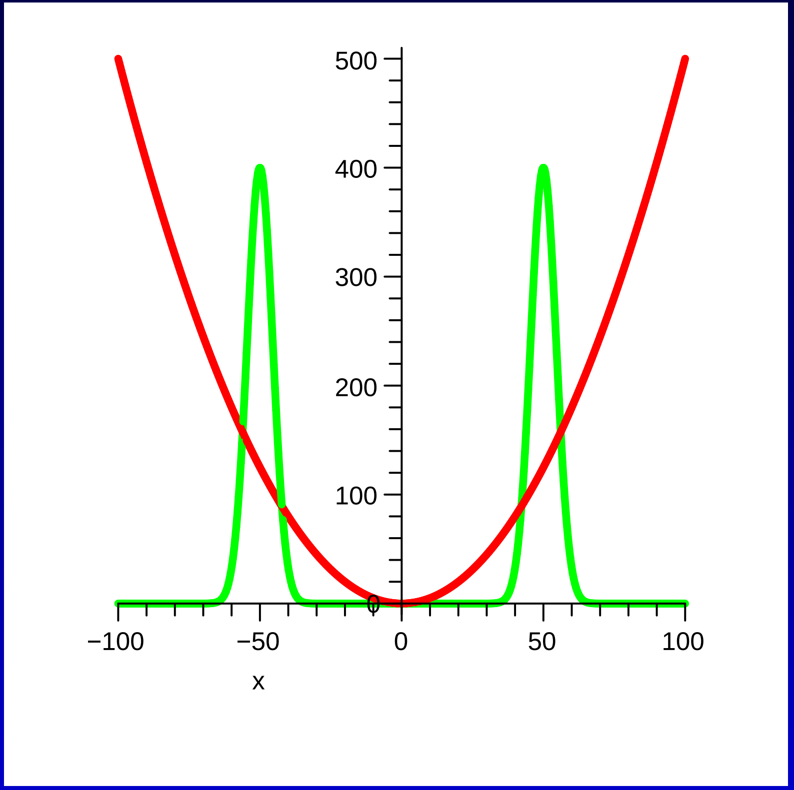

- Condensate allowed to evolve - trap drives components towards one another

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- If clouds have higher density then system is unstable with respect to soliton formation which leads, via a secondary instability, to the creation of vortex rings

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- Essential for phenomena such as depletion and the long time evolution of dynamically unstable systems
- Also need to account for thermal effects beyond the mean field.

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- Range of methods available, none entirely satisfactory (failure to conserve correct quantities, incorrect collective mode energies etc)

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- Focus here has been on Truncated Wigner Approximation (TWA)
- Requires parallel evolution of an ensemble of identical GP systems with different initial conditions, drawn from a random distribution determined by the initial (possibly mixed) quantum state
- Averages over ensemble provide condensate dynamics, variances etc provide information on quantum fluctuations.

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- Ideal Task Farm application

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- Good statistics requires 200 realizations
- Efficient algorithm for solution of GP dynamics (Crank-Nicholson) which respects conservation laws - need many processors but for relatively short times.

Quantum Fluctuations

Dipole Oscillations in Optical lattice

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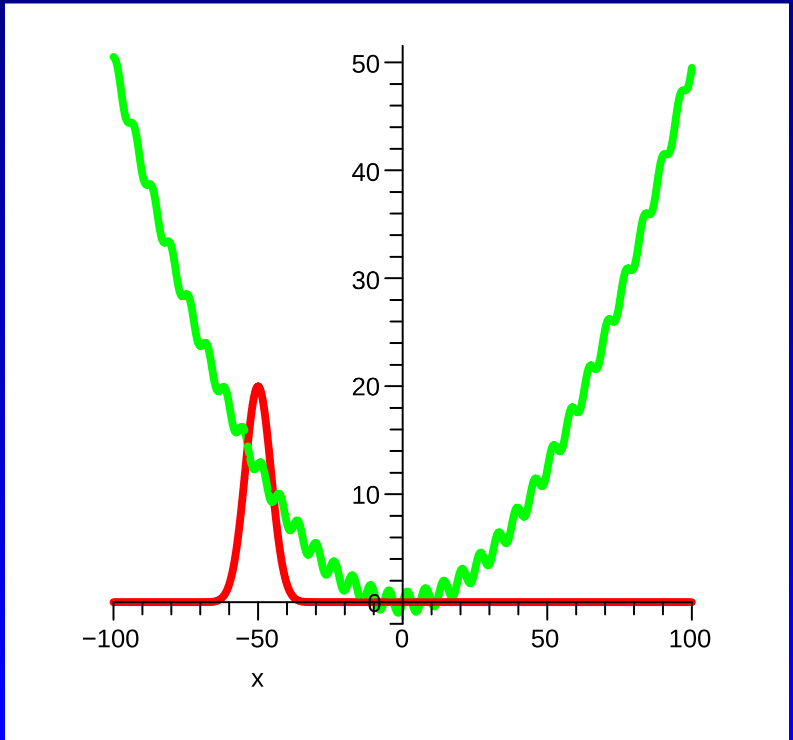
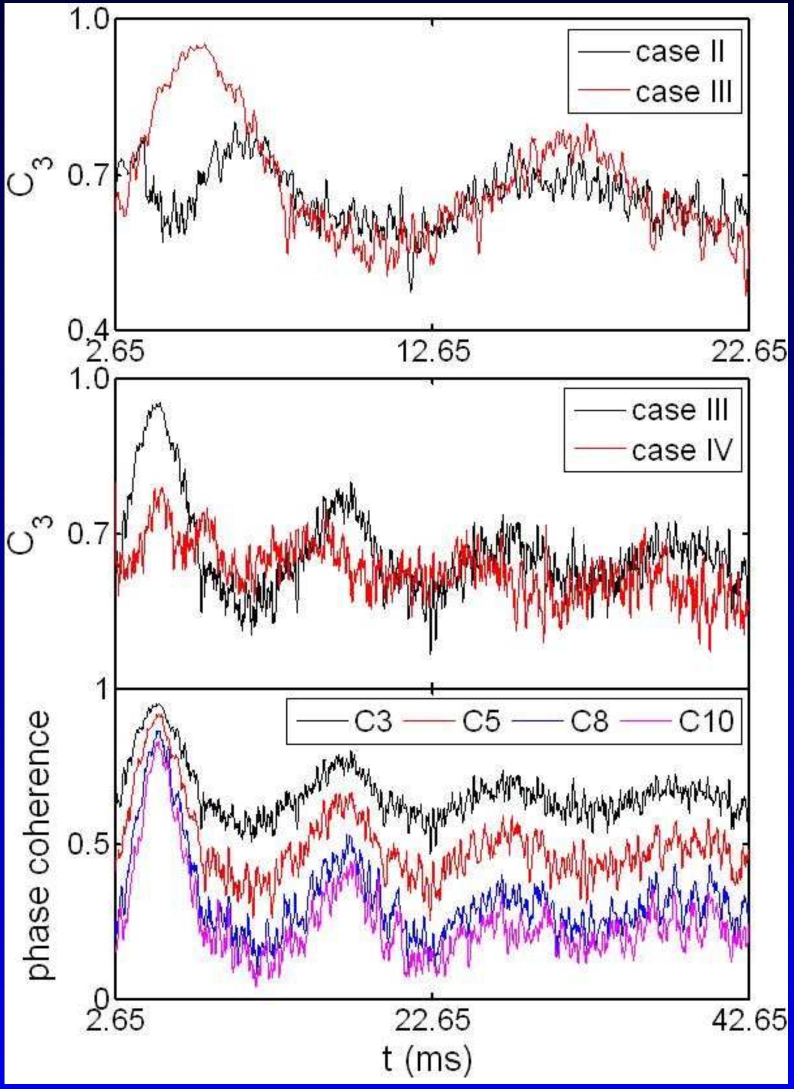
Dipole Oscillations in Optical lattice

- Consider a quasi 1d system in which a condensate is confined by a parabolic potential on which is superposed a periodic modulation
- Condensate is initially placed away from the centre of the trap and allowed to evolve.



Dipole Oscillations

Coherence Oscillation



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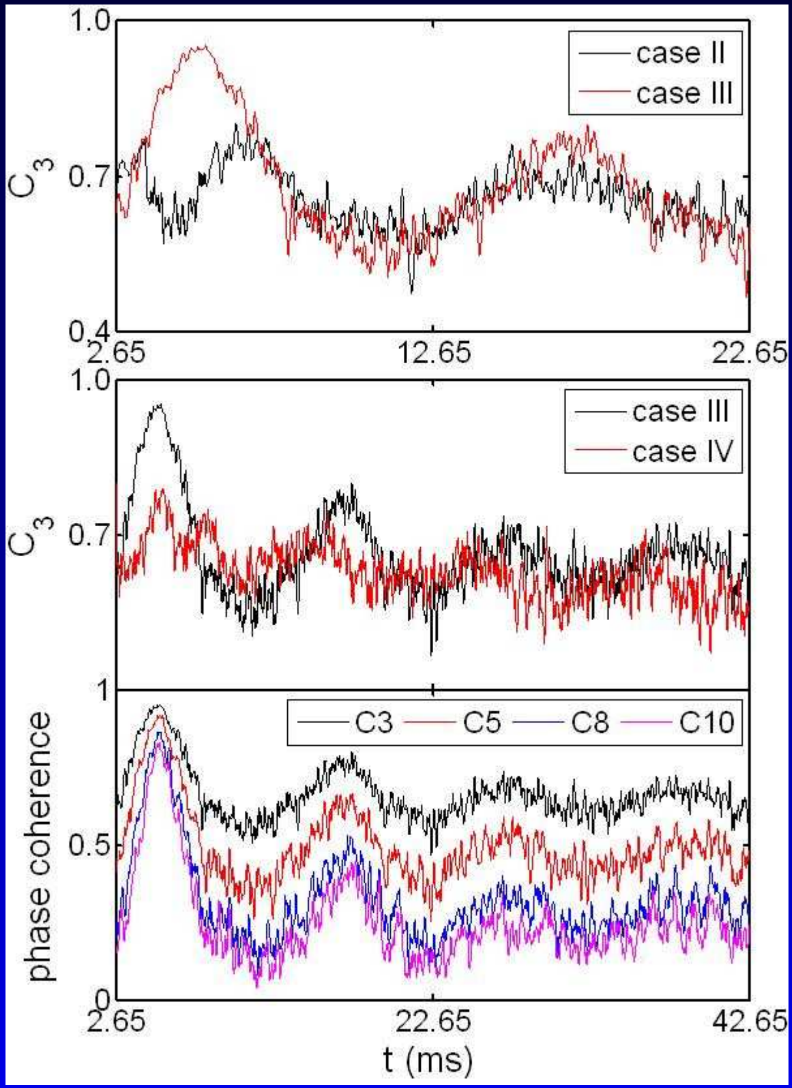
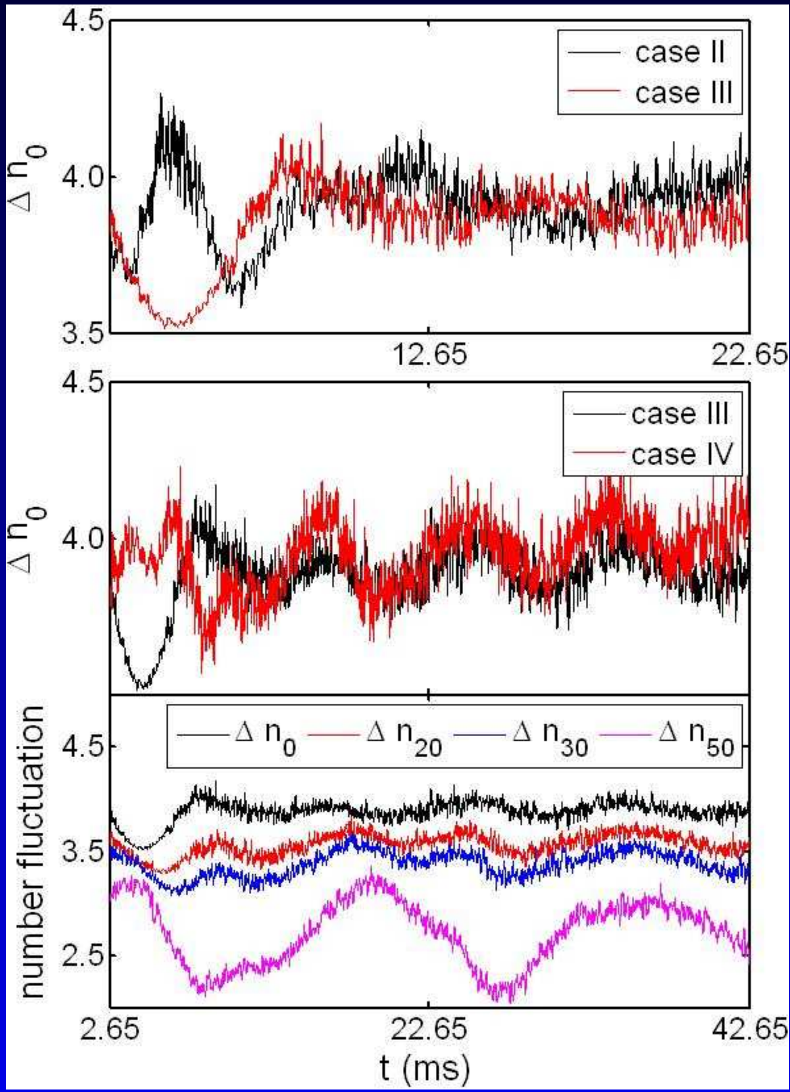


Number Fluctuations



Conclusions

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- Use of large task farm enables the investigation of quantum and thermal fluctuation effects in systems of ultra-cold Bosonic atoms, including number fluctuations and the investigation of decoherence.